Formelsammlung Regelungs- und Automatisierungstechnik

Markus Becker

16. Juli 2010

1 Time-Domain — Differential Equations

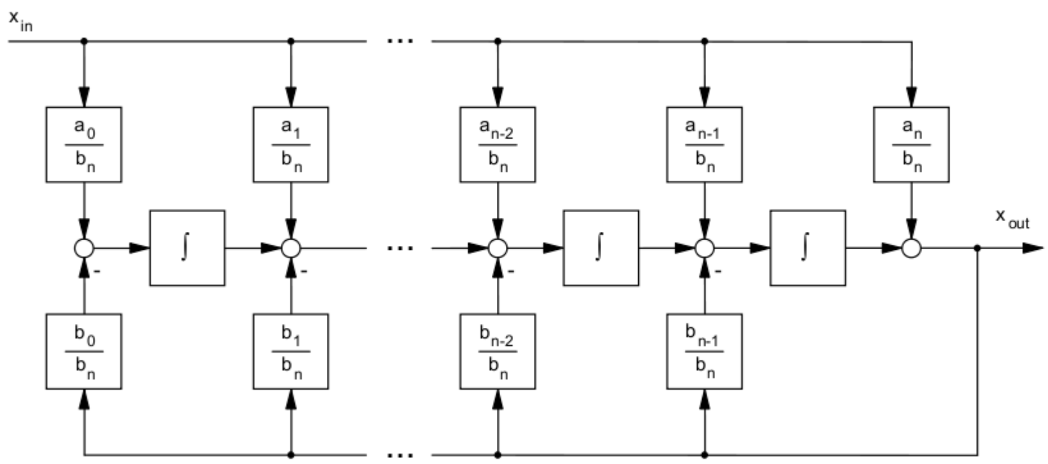
Solution	$x_{out}(t) = x_{out,t}(t) + x_{out,f}(t)$ with: $x_{out,t}$ = transient response after external stimulation $x_{out,f}$ = forced response part of the solution, describes steady state forced by continuous external stimulation
transient response part	$x_{out,t} = \sum_{i=1}^{n} K_i \cdot e^{\lambda_i \cdot t} \quad \vert \quad \lambda_i = \sigma_i + j\omega_i$

<u>Creation of a block diagram of a system from the differential equation</u>

Given the diff. eq.: $b_n \overset{(n)}{x}_{out} + \ldots + b_2\ddot{x}_{out} + b_1\dot{x}_{out} + b_0 x_{out} = a_m \overset{(m)}{x}_{in} + \ldots + a_2\ddot{x}_{in} + a_1\dot{x}_{in} + a_0 x_{in}$

(technical systems: m ≤ n).

Scheme for drawing the block diagram:



Hint: If m < n, all higher, non-existing a_μ are equal to 0.

2 Image-Domain

Frequency Response Function	$x_{in}(t) = cos(\omega t) + jsin(\omega t) = e^{j\omega t}$ $x_{out}(t)\vert_{t\to\infty} = x_{out,f}(t) \quad \vert \quad x_{out,f}(t) = A \cdot e^{j(\omega t+\varphi}$
example:	$T \cdot \dot{x}_{out} + x_{out} = K \cdot x_{in} \quad \vert \quad A \cdot e^{j\varphi} = \frac{K}{Tj\omega+1}$

initial- and final-value $x(t)|_{t\to 0} = \lim_{s\to\infty} sX(s)$ | $x(t)|_{t\to\infty} = \lim_{s\to 0} sX(s)$

2.1 Block Diagramm Manipulation

Chain $F_{series}(s) = F_1(s) \cdot F_2(s)$

Parallel $F_{paralles}(s) = F_1(s) + F_2(s)$

Loop $F_{Loop}(s) = \frac{F_f(s)}{1+F_f(s)\cdot F_r(s)}$ with negative Feedback

$F_{Loop}(s) = \frac{F_f(s)}{1-F_f(s)\cdot F_r(s)}$ positive Feedback

3 Combining Systems

3.1 Root-Locus-Plot

Applicable only if transfer-function is rational (no time-delays!)

- n branches start for $K = 0$ in the n poles of the open control loop
- m branches end for $K \to \pm\inf$ in m open-loop zeros
- $n - m$ branches extend for $K \to \pm\inf$ to infinity, tending to asymptotes:

$$s_{cg} = \frac{\sum_{\nu=1}^{n} s_{po_\nu} - \sum_{\mu=1}^{m} s_{zo_\mu}}{n-m} \quad | \quad \varphi_k = \frac{180°}{n-m} \cdot (2k-1) \quad | \quad \text{for } k = 1 \ldots (n-m)$$

- points are located on the real axis or symmetrically towards the real axis
- a point s is a point of the RLP, if there is an odd number of poles and zeros to the right of it

$$\frac{\prod_{\nu=1}^{n} |s - s_{po_\nu}|}{\prod_{\mu=1}^{m} |s - s_{zo_\mu}|} = |K \cdot Q|$$

3.2 Nyquist Stability Criterion

Small steady state control error $A(\omega) >> 1$ where $\omega << \omega|_{A=1}$

Stability $\omega|_{A=1} < \omega|_{\varphi=-180°}$
Phase margin: $\varphi_M = \varphi + 180°$ where $\omega = \omega|_{A=1}$
satisfying reaction on disturbance: $\varphi_M \geq 30°$
good command response (few oszillations): $\varphi_M \approx 60°$
good command response (overdamped): $\varphi_M \geq 80°$

Suppression of disturbances (measuring noise) on feedback-path $A(\omega) << 1$ where $\omega >> \omega|_{A=1}$

3.3 Ziegler-Nichols-Method

Controller	K_P	T_I	T_D
P	$0{,}5 \cdot K_{P,crit}$	-	-
PI	$0{,}45 \cdot K_{P,crit}$	$0{,}85 \cdot T_{crit}$	-
PID	$0{,}6 \cdot K_{P,crit}$	$0{,}5 \cdot T_{crit}$	$0{,}12 \cdot T_{crit}$

3.4 Dynamic Systems in State-Space

State-Space $\dot{\vec{x}}(t) = A\vec{x}(t) + B\vec{u}(t)$ | $\vec{y}(t) = C\vec{x}(t) + D\vec{u}(t)$
with $\vec{x}$ = state vector (n,1), $\vec{u}$ = input (control) vector (m,1), $\vec{y}$ = output vector (r,1)
A = state matrix (n,n), B = control (input) matrix (n,m), C = output matrix (r,n),
D = direct transmission (r,m)
with n = number of state variables, m = number of inputs, r = number of outputs

Transfer-Function	$F(s) = C \cdot (s \cdot I - A)^{-1} \cdot B + D$
Stability	Eigenvalues of the state-matrix A = poles of the system $det(s \cdot I - A) == 0$

3.5 Digital Control

Sample Period	$T_s \leq \frac{1}{10} \cdots \frac{1}{20} \cdot T_{system,main}$
z-Transform	from Laplace: $s \approx \frac{2}{T_s} \cdot \frac{z-1}{z+1}$ | $e^{-s \cdot T_s} = z^{-1}$